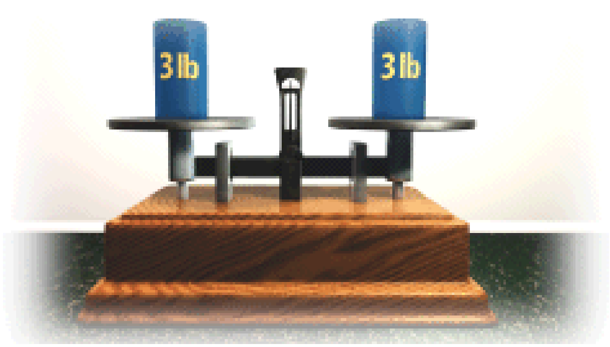
Student Learning Centre

Solving And Rearranging Equations

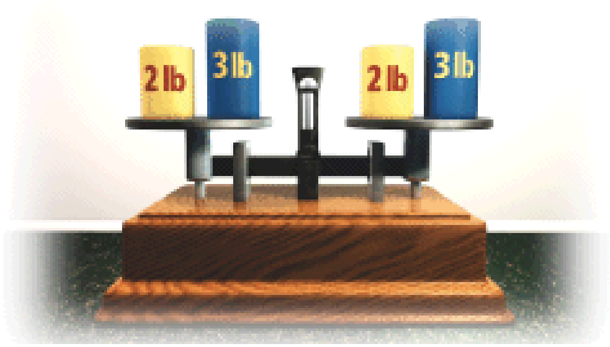
Flinders University — inspiring achievement

To ***solve*** an equation, we need to get the *x* by itself – when it's by itself, we get the answer of what *x* is! In order to get the *x* by itself, we need to ***rearrange*** the numbers and symbols in the equation while still keeping the equation accurate.

Think of the equals sign in the equation as a balance scale. We can change the positions of items on the scales, and take items on or off – we can change the position of numbers and symbols in the equation, and remove numbers or add them on – as long as we keep the scales balanced. We keep the scales balanced by ***always doing the same thing to both sides of the equation***.



$$\mathbf{3 = 3}$$



$$\mathbf{2 + 3 = 2 + 3}$$

Remember also that the sign of a variable or constant is what is in **front** of it; sometimes it's an "invisible +" if it's at the beginning.

To ***solve*** an equation, ***rearrange*** so that all variable parts (anything with x in) are on one side of the equal sign, and all number parts (parts with just numbers, not x's) are on the other side. To do this rearranging, you need to identify what operations are being used (Add, Subtract, Multiply, Divide) and "Undo" operations by using opposite operations. *Remember: Whatever you do to one side, you must do to the other side to keep equation balanced.*

Example 1: Solve 4x – 5 = 15

Our answer makes LHS = RHS in this equation so our answer is correct.

$4x - 5 = 15$ $+5 \quad\quad +5$	• Add 5 to both sides because this will remove the minus 5 from the LHS and leave just the $4x$ (add and subtract are opposite operations).
$4x = 20$	• Simplify – notice there's now only x parts on LHS and number parts on RHS.
$\frac{4x}{4} = \frac{20}{4}$	• Divide both sides by 4 because 4 is multiplied by x, so the opposite operation – division by 4 – will remove the 4 and leave only x .
$\boldsymbol{x} = \mathbf{5}$	• Simplify – notice we now have the x by itself and our answer is $x = 5$.

Let's check our answer in the original problem by replacing x with 5: $4 \times 5 - 5 = 20 - 5 = 15$.

The most important rule to remember is to ***do the same thing to both sides of the equation***. This preserves equality.

Example 2: Solve $x/3 + 4 = 9$

Our answer makes LHS = RHS in this equation so our answer is correct.

$\frac{x}{3} + 4$ -4	$=$	9 -4	• Subtract 4 from both sides because add and subtract are opposite operations, so subtracting 4 removes plus 4 from LHS and leaves just $\frac{x}{3}$.
$\frac{x}{3}$	$=$	5	• Simplify – notice there's now only x parts on LHS and number parts on RHS.
$\frac{x}{3}$ x 3	$=$	5×3	• Multiply by 3 on both sides because x is divided by 3, so the opposite operation – multiplication by 3 – will remove the 3 and leave only x .
$\boldsymbol{x}$	$=$	$\mathbf{15}$	• Simplify - notice we now have the x by itself and our **answer is x = 15.**

Let's check our answer in the original problem by replacing x with 15: $\frac{15}{3} + 4 = 5 + 4 = 9.$

Example 3: Solve $\frac{5+3x}{2} + 5 = 3x$

Our answer makes LHS = RHS in this equation so our answer is correct.

$\frac{5+3x}{2} + 5$ -5	$=$	$3x$ -5	• Subtract 5 from both sides because add and subtract are opposite operations. So subtracting 5 removes plus 5 from LHS and leaves just $\frac{5+3x}{2}$
$\frac{5+3x}{\cancel{2}} \times \frac{\cancel{2}}{1}$	$=$	$(3x-5) \times \frac{2}{1}$	• Multiply by 2 on both sides because 5+ $3x$ is divided by 2, so the opposite operation – multiplication by 2 – will remove the 2 and leave only 5+ $3x$. NOTE: $\frac{2}{1}$ is the same as 2, since 2 divided by 1 equals 2.
$5 + 3x$ $-3x$	$=$	$6x - 10$ $-3x$	• Simplify by multiplying LHS and expanding brackets RHS. Then subtract $3x$ from both sides since add and subtract are opposite operations, so subtracting $3x$ removes $+3x$ from LHS.
5 $+10$ 15	$=$ $=$	$3x - 10$ $+10$ $3x$	• Add 10 to both sides, removing -10 from RHS and rearranging equation with only x parts on LHS and number parts on RHS.
$\frac{15}{3}$	$=$	$\frac{3x}{3}$	• Divide both sides by 3 because 3 is multiplied by x, so the opposite operation will remove the 3 and leave only x.
5	$=$	x	• Our answer is: $x = 5$

Let's check our answer in the original problem by replacing x with 5: LHS: $\frac{5+3\times5}{2} + 5 = \frac{20}{2} + 5 = 15$, RHS: $3 \times 5 = 15$
Note both sides equal so answer is right.

Example 4: Solve $10y-(4y+8)=-20$			
$10y-(4y+8)$ $10y+(-1)(4y+8)$	$=$ $=$	-20 -20	• Distribute -1 on the left side.
$10y+(-1)(4y)+(-1)(8)$ $10y-4y-8$	$=$ $=$	-20 -20	• Simplify.
$6y-8$ $+8$	$=$	-20 $+8$	• Add 8 to both sides to get $6y$ by itself.
$6y$ $\frac{6y}{6}$	$=$ $=$	-12 $\frac{-12}{6}$	• Divide both sides by 6 to get y by itself.
$\boldsymbol{y}$	$=$	$\mathbf{-2}$	• **ANSWER**

Let's check our answer in the original problem by replacing y with -2:
LHS: $10\times(-2)-(4\times(-2)+8)=-20-(-8+8)=-20-0=-20$

Our answer makes LHS = RHS in this equation so our answer is correct.

Making a variable the subject of an equation

Sometimes a question asks you to make a variable the subject of an equation. This means you need to get a variable by itself on one side of the equals sign, so it's just like solving an equation. For example, if Q = 110 - $4P$, and you are asked to make P the subject of the equation, the way to do this is just to solve the equation – i.e. to get P by itself on one side of the equals sign.

Example 5: Make P the subject of $Q=110-4P$			
Q	$=$	$110-4P$	• Subtract 110 from both sides to get 4*P* by itself.
$Q-110$ $\frac{Q-110}{4}$	$=$ $=$	$4P$ $\frac{4P}{4}$	• Divide both side by 4 to get *P* by itself.
$\boldsymbol{\frac{Q-110}{4}}$ $\boldsymbol{P=\frac{Q-110}{4}}$	$=$ **OR**	$\boldsymbol{P}$	• **ANSWER**

Practice Questions

Solve:

1. $2x - 5 = 17$

2. $3y + 7 = 25$

3. $5n - 2 = 38$

4. Rearrange this formula **$A = 2a^2 + 4ab$** so that b is the subject of the formula.

5. **$s = ut + ½at^2$** is a formula used in Physics to calculate distance. Make **"a"** the subject of the formula.

Question 1:
$x = 11$

Question 2:
$y = 6$

Question 3:
$n = 8$

Question 4:

Subtract $2a^2$ from both sides:
$A - 2a^2 = 4ab$

Swap Sides:
$4ab = A - 2a^2$

Now, divide both sides by 4a:

$$\frac{4ab}{4a} = \frac{A - 2a^2}{4a}$$

$$b = \frac{A - 2a^2}{4a}$$

Question 5:

Subtract ut from both sides:
$\Rightarrow s - ut = ut + ½at^2 - ut$
$\Rightarrow s - ut = ½at^2$

Swap sides:
$\Rightarrow ½at^2 = s - ut$

Multiply both sides by 2:
$\Rightarrow 2 \times ½at^2 = 2 \times (s - ut)$
$\Rightarrow at^2 = 2(s - ut)$

Divide both sides by t^2
$\Rightarrow at^2 \div t^2 = 2(s - ut) \div t^2$

And we get: $$a = \frac{2(s - ut)}{t^2}$$

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